Faculty of Engineering | **Philadelphia University** **Mechanical Vibration (620414)** | Mechanical Eng. Dep.

Second Exam

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Student Name: | **Student ID number:**

Problem #1: chose the correct answer table fill the table with your choices. **(6marks)**

Answer	**Question**					
	1	**2**	**3**	**4**	**5**	**6**
a		X				X
b				X		
c	X		X			
d						
e					X	

1. Which of the following condition is correct for over damped system

a. $\zeta<1$ b. $\zeta=1$ c. $\zeta>1$ d. $\zeta=0$ e. none of the previous

2. Fill the blank: we can obtain a damping frequency when the system is ________

a. Under damped b. Critically damped c. Over damped d.Un-damped e. none of the previous

3. at resonance, which of the following statement is true

a. $r \ll 1$ b. $r < 1$ c. $r = 1$ d. $r > 1$ e. $r = 0$

4. Find the damping ratio (ζ) for single DoF system if k = 1000N/m, m=10kg and c = 100 N-s/m.

a. 1.00 b. 0.50 c.0.25 d.0.10 e. 0.05

5. The damping force in viscous damper depends on

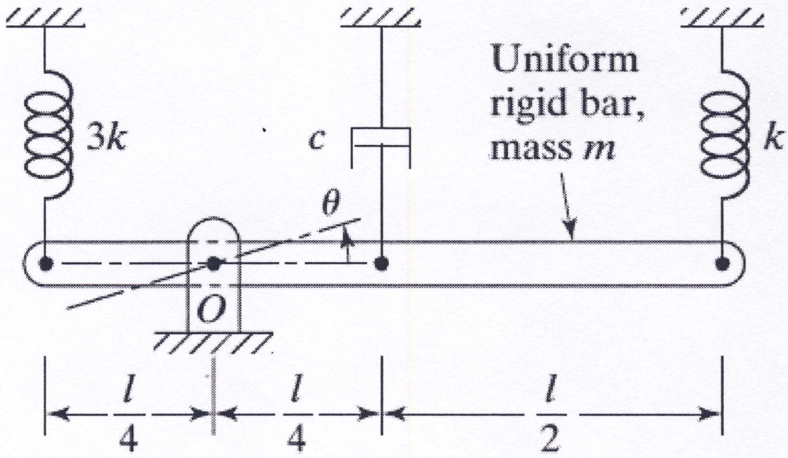
a. damping coefficient b. mass velocity c. mass acceleration d. a+c e. a+b

6. Find the critical damping (c_c) for single DoF system if k = 1000N/m, m=10kg and c = 100 N-s/m.

a. 200 N-s/m b. 100 N-s/m c.1000 N-s/m d.0.50 N-s/m e. 1 N-s/m

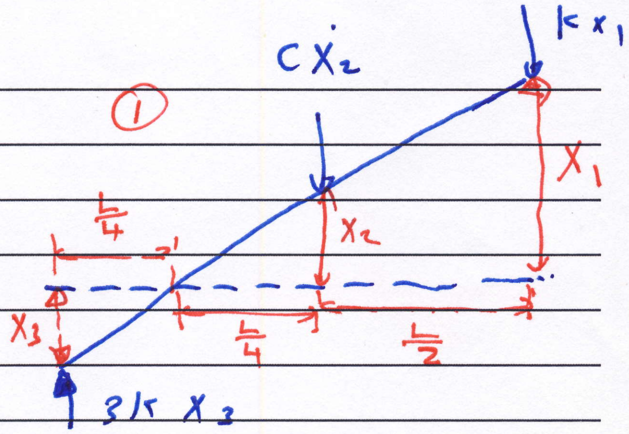
Problem #2: **(6 marks)**

1. **Draw the free-body diagram**
2. **Derive the equation of motion using Newton' s second law of motion for the systems shown in figure**
3. **Find its natural frequency in terms of system parameters**
4. **Assume $m_{Rod} = 9$ kg, $l = 2$m, c = 150 N-s/m and k=1000 N/m, find the damping ratio and the damping frequency (if it is possible)**



Assume small angular deflection, $J_O = \frac{7}{48} m_{Rod} l^2$

(2) Newton's 2nd Law.



$$\Sigma M_o = J_o \ddot{\theta}$$

$$J_o \ddot{\theta} = -3k X_3 \frac{L}{4} - C \dot{X}_2 \frac{L}{4} - k X_1 \frac{3L}{4}$$

Rearrange terms:

$$J_o \ddot{\theta} + 3k X_3 \frac{L}{4} + C \dot{X}_2 \frac{L}{4} + k X_1 \frac{3L}{4} = 0$$

From Sine Law:

$$\sin(\theta) \cong \theta = \frac{X_1}{3L/4} \Rightarrow X_1 = \left(\frac{3L}{4}\right) \theta \quad \text{---(1)}$$

$$X_2 = \left(\frac{L}{4}\right) \theta \quad , \quad X_3 = \left(\frac{L}{4}\right) \theta$$

$$\dot{X}_2 = \left(\frac{L}{4}\right) \dot{\theta}$$

Substitute in EoM:

$$J_o \ddot{\theta} + k\left(\frac{3L}{4}\right)\left(\frac{L}{4}\right)\theta + c\left(\frac{L}{4}\right)\left(\frac{L}{4}\right)\dot{\theta} + k\left(\frac{3L}{4}\right)\left(\frac{3L}{4}\right)\theta = 0$$

$\Rightarrow$

$$J_o \ddot{\theta} + \frac{cL^2}{16}\dot{\theta} + \frac{12kL^2}{16}\theta = 0$$ Ans.

(3) Natural Frequency ω_n:

$$\omega_n = \sqrt{\frac{12L^2}{16}k / J_o} \quad ; J_o = \frac{7}{48} M_{rod} L^2 = 5.25 \ kg.m^2$$ ans

$$\Rightarrow \omega_n = \sqrt{\frac{12}{16}(2^2)1000 / 5.25} = 23.9046 \ rad/sec$$ ans

(4-a) Damping ratio (ζ)

$$\zeta = \frac{c}{c_c} = \frac{c_{eq}}{2m_{eq}\omega_n} \quad ; m_{eq} = J_o \Rightarrow \zeta = \frac{cL^2/16}{2J_o \omega_n}$$

$$\zeta = \frac{150(2)^2/16}{(2)(5.25)(23.9046)} = 0.1494$$ Underdamped

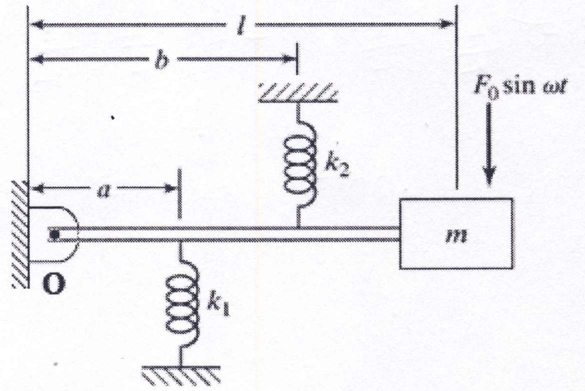
(4-b) Damping Frequency: ω_d

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n = \sqrt{1-0.1494^2}\,(23.9046)$$

$$\omega_d = 23.6363 \ rad/sec$$ ans.

Problem #3: if the system shown in the figure represents a **rod** connected to linear springs (k_1 and k_2) **(6marks)**

If the **rod** has the following parameters:



1. $a = \frac{l}{3}, b = \frac{2l}{3}, l = 1m, m_{Rod} = 3kg$
2. m = 5 kg
3. $J_O = \frac{1}{3} m_{Rod} l^2 + ml^2$

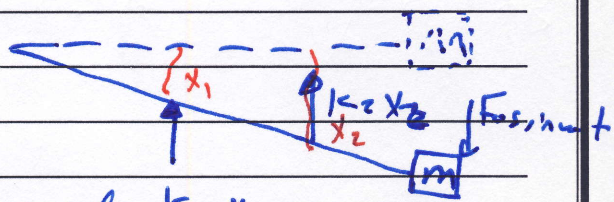
And

1. the stiffness of each spring is: $k_1 = 5000$ N/m and $k_2 = 3000$ N/m
2. $F_o = 100$N and $\omega = \pi$ rad/s.

1. Draw FBD for this system
2. Derive the equation of motion and find the natural frequency of this system
3. Find the magnification factor.

Assume small angular deflection

EoM + ω_n



$\sum M_o = J_o \ddot{\theta}$

$J_o \ddot{\theta} = -k_1 x_1 a - k_2 x_2 b + F_o \sin(\omega t) L$ $k_1 x_1$

$J_o \ddot{\theta} + k_1 x_1 a + k_2 x_2 b = F_o L \sin(\omega t)$

Apply sine Law.

$x_1 = a\theta$, $x_2 = b\theta$

Substitute in EoM:

$J_o \ddot{\theta} + k_1 a^2 \theta + k_2 b^2 \theta = F_o L \sin(\omega t)$

$\Rightarrow J_o \ddot{\theta} + (k_1 a^2 + k_2 b^2)\theta = F_o L \sin(\omega t)$

$\omega_n = \sqrt{\frac{k_1 a^2 + k_2 b^2}{J_o}}$, $a = \frac{L}{3} = \frac{1}{3}$, $b = \frac{2L}{3} = \frac{2}{3}$

$J_o = \frac{1}{3} m_{rod} L^2 + m L^2 = \frac{1}{3}(3)(1)^2 + (5)(1)^2 = 6 \text{ Kg.m}^2$

$$\omega_n = \sqrt{\left[(5000)\left(\tfrac{1}{3}\right)^2 + 3000\left(\tfrac{2}{3}\right)^2\right] / 6} = 17.743 \text{ rad/sec}$$

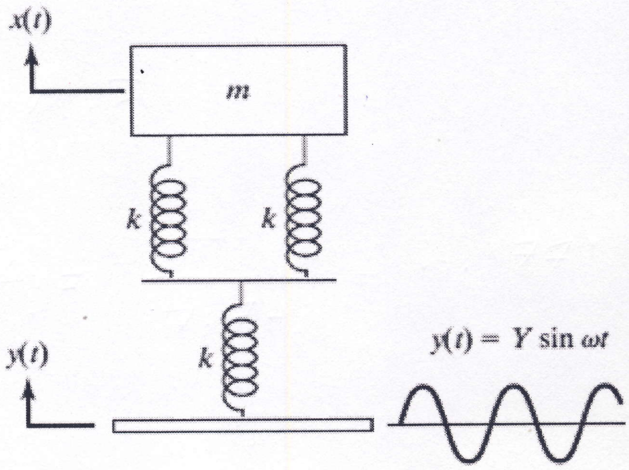
3) Magnification Factor: M.F

$$M.F = \frac{1}{1-r^2}, \quad r = \frac{\omega}{\omega_n} = \frac{\pi}{17.743} = 0.1771$$

$$M.F = \frac{1}{1-(0.1771)^2} = 1.0324.$$

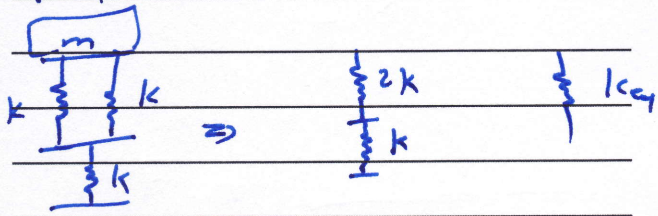
Problem #4: if the system shown in the figure represents mass connected to linear springs and undergo base excitation. **(4marks)**

1. **Derive the governing equation**
2. **Derive the expression for the natural frequency**



① E.O.M

Keq :



$$k_{eq} = \frac{2k}{3}$$

$$\sum F = m\ddot{x}$$

$$m\ddot{x} + \frac{2k}{3}(x-y) = 0$$

$$m\ddot{x} + \frac{2k}{3}x = Y\sin(\omega t)$$ Ans.

② ω_n :

$$\omega_n = \sqrt{\frac{2k}{3}/m}$$ Ans